A robust, discrete, near time-optimal controller for hard disk drives

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Abstract

This paper presents a robust, discrete, near time-optimal control algorithm for hard disk drives. Building on the continuous-time control law of Newman [Trans. Automatic Control 35 (7) (1990) 841] and Newman and Souccar [J. Dynamic Syst. Measure. Control 113 (1991) 363], the chattering phenomenon for continuous-time systems is first examined, and it is shown how additional chattering can occur in the discrete-time case. A criterion for the selection of control parameters to prevent chattering in the discrete-time case while assuring controller stability is then suggested. The controller developed is applied to a commercial hard disk drive. Compared to the more commonly used PTOS, the proposed controller significantly reduces chattering while maintaining comparable seek time to reach the destination track.

Keywords: Robust; Discrete; Near time-optimal control; Sliding mode control; PTOS; Hard disk drive

1. Introduction

The two main functions of a disk drive read/write (R/W) head positioning system are track-seeking and track-following. From a control perspective, track-seeking can be viewed as a time-optimal control problem in which the objective is to move the R/W head from the current track to a specified destination track in minimum time using a bounded control effort. Once the head is in the neighborhood of the destination track, the servo system switches from the track-seeking mode to the track-following mode to accurately position the head on the track center.

An analytic solution to the time-optimal control can be obtained in a number of cases. However, physical realizations typically exhibit chatter or divergence due to parameter uncertainty resulting from manufacturing tolerance, temperature variations and aging effects. Hence, robustness to parameter variations is essential for time-optimal control to be successfully applied to physical systems.

Among the extensive literature on time-optimal control, Newman [1] suggests a nonlinear control technique for achieving robust, nearly time-optimal control of a scalar double integrator plant. This technique involves the combination of the traditional “bang-bang” time-optimal control with methods of sliding-mode control. Newman & Souccar [2] present a technique for controlling a scalar second-order nonlinear system using a combination of bang-bang time-optimal control, sliding-mode control, and feedback linearization. The control laws in [1] and [2] have two distinctive properties. First, the step function that occurs in the solution of the time-optimal control problem is replaced by a saturation function whose continuity properties can be exploited to eliminate chattering. Second, techniques from sliding-mode control and PD control are combined with time-optimal control in such a way that the desired trajectory can be tracked in the presence of parameter uncertainties and linear control effort is exerted in the neighborhood of the destination.

In this paper, the two main causes of chattering for a scalar double-integrator plant are first examined. In particular, it is shown that additional chattering phenomena not present in the continuous-time case can occur in the discrete-time. To eliminate these chattering effects, the existing continuous-time, robust time-optimal control is extended to the discrete time case by suggesting a criterion for the selection of control parameters. This criterion also prevents chattering while guaranteeing stability. The performance of this controller is then experimentally verified by applying it to a commercial hard disk drive (HDD).

With respect to the hard disk drive control literature, the most popular controller currently being used in commercial hard disk drives is the so-called proximate time-optimal servomechanism (PTOS) as presented by Franklin et al. [3], Workman et al. [4], Lee et al. [5] and, Weerasooriya et al. [6]. Like Newman and Souccar’s controller, PTOS also substitutes the step function with the saturation function to eliminate chattering and exerts linear control effort in the neighborhood of the destination. However, both PTOS and New-
man and Souccar’s controller are obtained using rather different approaches in the decelerating region. In PTOS, the control input is proportional to the difference between the current velocity and the pre-designed velocity trajectory, while Newman and Souccar’s controller invokes sliding-mode control to make the system track the desired near time-optimal decelerating trajectory. This paper presents an alternative to the PTOS widely used in the HDD industry. PTOS divides the phase plane into three regions, i.e., linear control using PD (region III), saturation bang-bang control (region I) and unsaturated transition (region II) between the first two states (see Fig. 2 later in text). This paper proposes a control law for region II, and it can be viewed as an extension of reference [2].

The paper is organized as follows. Section 2 analyzes the cause of the chattering problem when traditional “bang-bang” controllers are used, and introduces Newman’s robust near time-optimal control as a means of overcoming this problem. Section 3 presents a discrete-time version of Newman’s robust near time-optimal controller, and a criterion for selecting control parameters to prevent chattering in the discrete-time case. In Section 4, a hard disk drive and its mathematical model is described, a state estimator is designed for experimental purposes, and the controller is applied to the hard disk drive to evaluate its performance against simulation results.

2. Robust near time-optimal control : continuous-time case

Analytic solutions to the time-optimal control problem for a double integrator plant can be derived as follows. First, let the system be

\[ J_t \ddot{x} = u, \quad |u| \leq u_{\text{max}} \]  

(1)

where \( J_t \) is a constant scalar and \( u_{\text{max}} \) is the control bound. The objective is to find the optimal \( u \) that minimizes

\[ \min_u \int_{t_0}^{t_f} dt \]  

(2)

subject to the state equations (1), and boundary conditions

\[ x(t_0) = x_0, x(t_f) = x_f, \]  

where \( x_0 \) is fixed and \( t_f \) is free. As it is well known, the solution is given by

\[ u = -u_{\text{max}} \text{sgn}(S) \]  

(3)

\[ S = x - x_f + \frac{J_t}{2u_{\text{max}}} \dot{x} \]  

(4)

where \( S \) is called the switching curve (Bryson and Ho [10], Kirk [11], Slotine and Li [12]).

In the event that the identified value of \( J_t \) is different from its true value, chattering can occur when the controller of (3) is applied. Consider, for example, the case where \( x_0 =\]
Newman [1] suggests a robust near time-optimal control that applies to the carriage deceleration problem except for the switching curve in the solution of the time-optimal control problem. The desired deceleration trajectory is driven in the same manner, thereby causing chattering as before.

To eliminate the chattering problem as discussed above, Newman [1] suggests a robust near time-optimal control that applies the switching function \( S(x, i) = 0 \), region I is defined as the states for which \( |x| > S_{sat} \). In this region, traditional bang-bang control is invoked because chattering does not occur. Observe that region I consists of 2 topologically disconnected regions. Region III is defined as a convex region about the goal state with boundaries \( |x| \leq S_{sat} \). Region III is defined as the states for which \( |x| > S_{lim} \), a sliding-mode controller is invoked to track the near-time-optimal trajectory without chattering. The exact form of the control law is given as follows:

\[
\begin{align*}
  u &= \begin{cases} 
    -u_{max} \operatorname{sgn}(S) & \text{for} |x| > S_{sat} \\
    -u_{max} \operatorname{sat} \left( \frac{1}{J_t} \left( \frac{x - S_{sat}}{S_{lim}} \right) + K_s \operatorname{sat} \left( \frac{S}{S_{sat}} \right) \right) & \text{for} |x| \leq S_{sat} \text{ and } |x| > S_{lim} \\
    -u_{max} \operatorname{sat} \left( K_v \operatorname{sat} \left( \frac{x - S_{sat}}{S_{lim}} \right) + K_s \operatorname{sat} \left( \frac{S}{S_{sat}} \right) \right) & \text{for} |x| \leq S_{sat} \text{ and } |x| \leq S_{lim}
  \end{cases}
\end{align*}
\]

In Eq. (6) the outermost "sat" restricts the absolute value of the control effort to be within \( u_{max} \). The various control parameters of Eq. (6) can be chosen according to the stability and performance criteria as suggested by Newman & Soucar [2].

3. Robust near time-optimal control: discrete-time case

In this section a discrete-time version of Newman and Soucar’s controller is first developed and, a set of criteria for selecting the control parameters of the discrete-time controller that ensures stability while eliminating chattering is proposed. This process uncovers a type of chattering phenomenon that is unique to the discrete-time case. Determination of the criterion for control parameter selection is guided by the following requirements. First, the control law must transfer the state from region I to region II or directly to region III. Finally, once region III is entered, the system should remain within region III until convergence to the goal state is achieved.

At first a discrete-time version of the continuous-time, robust, near time-optimal control of the previous section is developed. The states are defined as position and velocity. One has the following discrete-time, state-space description of the system:

\[
\begin{align*}
  \mathbf{x}(k+1) &= \begin{bmatrix} 1 & T_s/J_t \\ 0 & 1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} T_s^2/2J_t \\ T_s/J_t \end{bmatrix} u(k)
\end{align*}
\]
\begin{equation} |J_s - J_{\text{actual}}| \leq \Delta J_s \tag{9} \end{equation}

where \( J_s \) is an identified value of the true parameter \( J_{\text{actual}} \) and \( \Delta J_s \) is an upper bound on the modeling error. The control structure is a discrete-time mapping of the same robust, time-optimal control law used in continuous time. The mapped control law is as follows:

\[
u(k) = \begin{cases} -u_{\text{max}} \text{sign}(S(k)) & \text{for } |S(k)| > S_{\text{sat}} \\ u_{\text{max}} \text{sat} \frac{1}{u_{\text{max}}} \{ u_{\text{eq}} \text{sat} + K_s \text{sat} \frac{x_2(k)}{x_{\text{lim}}(k)} + K_v \text{sat} \frac{S(k)}{S_{\text{sat}}(k)} \} & \text{for } |S(k)| \leq S_{\text{sat}} \end{cases}
\]

where \( S(k) = x_1(k) - x_{\text{final}} + (J_s + \Delta J_s)/(2u_{\text{eq}}x_2(k)x_2(k)) \). The five control parameters to be determined in control law (10) are as follows:

\[
u_{\text{eq}} = \begin{cases} \frac{J_s}{J_s + \Delta J_s} & \text{for } |S(k)| > S_{\text{sat}} and |x_2(k)| > x_{\text{lim}} \tag{10} \\ J_s + \Delta J_s & \text{for } |S(k)| \leq S_{\text{sat}} and |x_2(k)| \leq x_{\text{lim}} \end{cases}
\]

In the continuous-time case, once the system is in region I, it can only enter region II or region III, but in the discrete-time case, states in the lower part of region I can move to the upper part of region I after one sampling time and vice versa (see Fig. 4), resulting in chattering. A condition which ensures that the system cannot “leap” across region II. Assuming \( x_2 > 0 \), one can calculate how long the system can move for a single sampling time interval:

\[
S(k+1) = x_1(k+1) - x_{\text{final}} + \frac{J_s}{2\nu_{eq}} |x_2(k+1)|^2
\]

Observe that the first term on the right side of the upper equation is equal to \( S(k) \). For convenience, define

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Escape from region II due to an underestimated \( J_s \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Illustration of unstable behavior.}
\end{figure}
The criterion for the selection of the control parameters is summarized in Table 1. The estimation of \( J_i \) and the maximum control effort \( u_{\max} \) maximize the difference in velocity. Thus, the width of region III must be larger than the maximum difference in velocity, i.e.,

\[
x_{\lim} \geq \frac{T_i u_{\max}}{2 (J_i - \Delta J_i)} \geq x_{2}(k) - x_{1}(k)
\]

Choosing \( x_{\lim} \) to be as small as possible will in general result in minimum seek time. If \( K_i \) is much larger than \( u_{\max} \), then the control law invoked in region II is similar to "bang-bang" control by reason that the width of the region where the control effort is under its maximum value becomes smaller. In such a case, chattering by leaps across region II can occur. \( K_i \) must therefore be upper bounded so that the region where the control effort is under its maximum value is wider than region II, i.e.,

\[
S_{\text{sat}} \leq S^+ - S^- = \left[ \frac{S_{\text{sat}}}{K_i} (u_{\max} - u_{eq}) \right] - \left[ \frac{S_{\text{sat}}}{K_i} (u_{\max} + u_{eq}) \right]
\]

where \( S^+ \) is defined as the the value of \( S(x_{1}(k), x_{2}(k)) \) for \( u = u_{\max} \) in region II and \( S^- \) is defined for \( u = -u_{\max} \). The upper bound on \( K_i \) is thus given by

\[
K_i \leq 2 u_{\max}
\]

The criterion for the selection of the control parameters derived in this section is summarized in Table 1. Fig. 6 shows the

**Fig. 5. Trajectory with a given initial state and maximum acceleration control effort.**

\[
\Delta S \triangleq S(k+1) - S(k) = T_i x_{2}(k) + \frac{T_i^2 u_{eq}^2}{2 J_i} + \frac{u_{eq}^2}{2 u_{eq} J_i} (12)
\]

If the width of region II, \( S_{\text{sat}} \), is larger than the maximum allowable movement for one sampling time, leaps across region II can be avoided. The larger the value of \( x \) and \( x_{2}(k) \), the larger is the value of \( \Delta S \). The control input \( u \) is bounded, and one can obtain the maximum value of the velocity \( V_{\max} \) in the neighborhood of the switching curve by referring to Fig. 5. Here \( V_{\max} \) is the intersection of the switching curve and the system trajectory for the given initial state and maximum control effort \( u_{\max} \). \( V_{\max} \) can be obtained from the following expression:

\[
V_{\max} = \sqrt{\frac{2 (S_{\text{sat}} - S_{\text{initial}})}{J_i - \Delta J_i} \left( \frac{u_{\max} u_{eq}}{u_{eq} + u_{eq}} \right)}
\]

Finally, chattering can be avoided by choosing \( S_{\text{sat}} \) as follows:

\[
S_{\text{sat}} \geq T_i V_{\max} + \frac{u_{\max} T_i V_{\max}}{u_{eq}} + \frac{u_{\max} T_i^2}{2 (J_i - \Delta J_i)} + \frac{u_{\max}^2}{2 u_{eq} (J_i - \Delta J_i)}
\]

The choice for \( x_{\lim} \) can be made in a similar fashion to that for \( S_{\text{sat}} \). The width of region III, \( 2 x_{\lim} \), must be larger than the maximum difference of the velocity over one sampling time interval. If not, chattering between the upper and the lower parts of region II can occur (see Fig. 4). The maximum over-

**Fig. 6. Schematic diagram for robust, near time-optimal control.**
schematic diagram for the proposed robust, near-time-optimal control. Referring to the figure, \( y_r(k) \) denotes the desired position, \( y(k) \) the current position, \( e(k) \) the position error, \( u(k) \) the control input, \( \dot{e}(k) \) the estimated velocity and \( u_{\text{max}}, u_{\text{min}} \) the maximum and minimum control input respectively.

4. Application to HDD

4.1. HDD actuator model

Yamaguchi et al. [7] give a concise and detailed introduction to hard disk drives and their control. Fig. 7 shows a basic schematic diagram of a head disk assembly (HDA). The commercial disk drive used for both simulation and experiment is a 3.5 in. type, with a size of 2.5 GByte and 6980 tracks per inch. The assembly configuration is as follows. Several disks are stacked on the spindle motor shaft, which rotates at 5400 rpm. Each disk surface contains thousands of data tracks, and a magnetic head is supported by a suspension and carriage that flies several micro-inches above the disk surface. The actuator, called a voice coil motor (VCM), actuates the carriage and moves the head onto the desired track. The mechanical part of the plant consists of the VCM, carriage, suspension, and heads. The controlled variable is the head position. Typically, on the back of the HDA, there exists a circuit board where a microprocessor or DSP for servo control is mounted. The position of the head is detected by the sector servo method as presented by Franklin et al. [3], and the sampling frequency in our experimental test bed is 5960 Hz.

The system considered in both simulation and experiments can be closely approximated by a pure double-integrator plant with control input saturation. It is assumed that there is uncertainty associated with the inertia. The continuous-time dynamics of a VCM can be described as follows:

\[
J_t \ddot{x} = u, \quad |u| \leq u_{\text{max}} \tag{17}
\]

where \( J_t \) is the inertia of the carriage, and \( x \) is the position of the head. Define the parameter error as follows:

\[
|J_t - \hat{J}_t| \leq \Delta J_t \tag{18}
\]

where \( J_t \) is the identified value of the true parameter \( J_t \), and \( \Delta J_t \) is an upper-bound on the modelling error which is used to design a robust controller. The following equivalent discrete time model of the actual system, is used for simulation and experiments

\[
x(k + 1) = \begin{bmatrix} 0 & 1 & T_e \\ 0 & 1 & \frac{T_e^2}{2J_t} \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ \frac{T_e}{J_t} \end{bmatrix} u(k) \\
y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) \tag{19}
\]

where \( x \triangleq [x_1 \ x_2]^T \), \( x_1 \) is the position of the R/W head, \( x_2 \) the velocity of the head, and \( T_e \) is the sampling period (\( = 1/5960 \) s).

4.2. State estimator

Since the head position \( x_1(k) \) is the only available measurement, the remaining state \( x_2(k) \) must be estimated through a state estimator prior to setting the control. As claimed by Franklin et al. [3], the current estimator is a better alternative than a predictive estimator, which provides the fastest response to unknown disturbances or measurement error and thus better regulation of the desired output. The current estimator based on the model of

![Fig. 7. Basic schematic diagram of head disk assembly.](image)

![Fig. 8. Experimental setup.](image)
Eq. (19) is given by

\[
\dot{z}_2(k) = (1 - L_c T_s) \dot{z}_2(k - 1) + \left( \frac{T_s}{T} - \frac{T_s^2 L_c}{2 L_m} \right) u(k - 1) \\
- L_c x_1(k - 1) + L_c x_1(k)
\]

(20)

The estimated velocity of the head, \( \dot{z}_2(k) \), obtained by Eq. (20) replaces the actual velocity of the head, \( z_2(k) \), in Eq. (10) and will be used in the next section describing simulation and experimental results.

4.3. Simulation and experimental results

Fig. 8 shows the experimental setup of the commercial HDD control test bed. The maximum value of the control input (which is the current) of this particular HDD is 0.5 A and
the inertia of the carriage is measured to be \(8.52 \times 10^4\) A s\(^2\)/m. Other specifications—rotating speed of the disk, track per inch, sampling period—are the same as those given in Section 4.1. To benchmark the performance of the proposed controller with that of the commonly used PTOS, a particular PTOS controller is briefly described. In order to implement PTOS control [3], four parameters need to be determined: the proportional gain \(k_1\), the differential gain \(k_2\), the linear region \(y_l\) where the linear PD controller is applied, the acceleration discount factor \(0 < \alpha < 1\) which allows us to accommodate uncertainties in the plant accelerating factor at the cost of an increase in the response time. To ensure the continuity
of the velocity profile, the following relation must hold [4]:

$$a = \frac{2k_1}{k_2 L_e}$$

(21)

For the PTOS controller, the gains $k_1$ and $k_2$ are determined so that the closed loop system has a natural frequency $\omega_n = 250\text{Hz}$ and damping factor $\gamma = 0.9$. Following the criterion suggested in Section 3, the parameters of the proposed controller are selected as follows:

$$u_{eq} = 0.45A, \quad S_{eq} = 1.5 \times 10^{-3}, \quad \dot{x}_{max} = 0.1, \quad K_1 = 0.954, \quad K_2 = 0.1, \quad L_e = 3564$$

The estimator gain, $L_e$, is calculated by choosing the desired pole of the estimator to be at 0.4. Throughout the simulation study with the PTOS and the proposed algorithm, two cases with underestimated and overestimated inertia (by a factor of 0.9 and 1.1, respectively) are considered. With these values for the control parameters the simulations of the PTOS and the proposed control are performed using Matlab 5.2.

Figs. 9 and 10 show 2300 track (one-third stroke move) seek profiles and the corresponding control inputs, respectively. The PTOS controller exhibits overshoot in the position trajectory and control input when the inertia is underestimated.

Fig. 11 shows the position of the head under the application of the control input shown in Fig. 12, which clearly suffers from neither chattering nor overshoot. The seek time is approximately 6 ms, which is about the same as that obtained by using PTOS embedded in the commercial HDD. In Figs. 11 and 12 it is seen that the seek time for the experiment is longer than that for the simulation, and the control input in the experiment approximates to $-0.45A$ in the region of deceleration while in the simulation it reaches the maximum value of $-0.5A$. The value of the deceleration control input in the experimental results of Fig. 12 indicates that the estimated inertia is closer to the true value, so that no large reserved control input is needed for tracking the desired deceleration trajectory. If one can estimate the inertia of the swing arm more accurately, the larger reserved control input for the sliding-mode correction becomes unnecessary and one can choose $u_{eq}$ closer to $u_{max}$, resulting in a shorter seek time.

5. Conclusion

In this paper the continuous-time, robust, near time-optimal control first proposed by Newman [1] and Newman and Soucare [2] was extended to the discrete-time case. The two main causes of chattering are first examined, and it is shown that chattering phenomena not present in the continuous-time controller can occur in the discrete-time case. Therefore, the robust, near time-optimal control is applied to the discrete time case in such a way that a criterion is developed for the selection of control parameters that prevents chattering while guaranteeing stability. The controller is tested on a commercial HDD to evaluate its performance against those predicted by simulation. Both computer simulation and experimental results show that the system does not suffer from chattering while achieving near time-optimal control. Disk drives typically have nonlinear dynamics, associated with the bias needed to cancel the effects of forces from the flex cable, gravity, friction at the head/disk interface, aerodynamics forces, and the pivot ball bearing of the rotary actuator of the disk drive (Eddy et al. [8], Ishikawa and Tomizuka [9]). These nonlinear dynamics effects should become more pronounced as the trend to higher capacity of disk drives with even narrower tracks continues. Although this paper only considers a double integrator plant, the control technique discussed here is applicable to a larger class of nonlinear systems, because a switching curve—interpreted as a deceleration trajectory—can be calculated in the same manner as the time-optimal control and thus can be used in more complex models of disk drives.

In practice, most actuators have a real axis pole (may be at very low frequency) and show high frequency resonance. The experimental results show most probably a resonant behaviour. However, this is not a resonance in the traditional sense, and the relatively small spikes mean that at the chosen sample rate the discrete controller exhibits relatively meager disturbance rejection. As shown in [3], for the case where sampling is slower than 10 times the bandwidth, degradation due to the discrete nature of the control over that possible with a continuous control is quite significant. Except for the case of controllers with small word size, performances continue to improve as the sample rate increases, even though performance reductions have a tendency to appear for sampling faster than 40 times the bandwidth. Choosing the adequate sampling rate is a trade-off between errors generated by plant disturbances, cost and eventual resonances in the plant that are faster than the bandwidth. This suggests further study on the proposed controller, i.e., identification of the servo bandwidth, gain crossover, frequency, and stability margins. Another direction for future research is to investigate how the proposed controller responds to a repeatable and non-repeatable run out disturbance as well as uncertainty in the model, and perform simulation and experiments using a non-double integrator plant.

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